EE 232: Lightwave Devices
Lecture #5 – Quantum well

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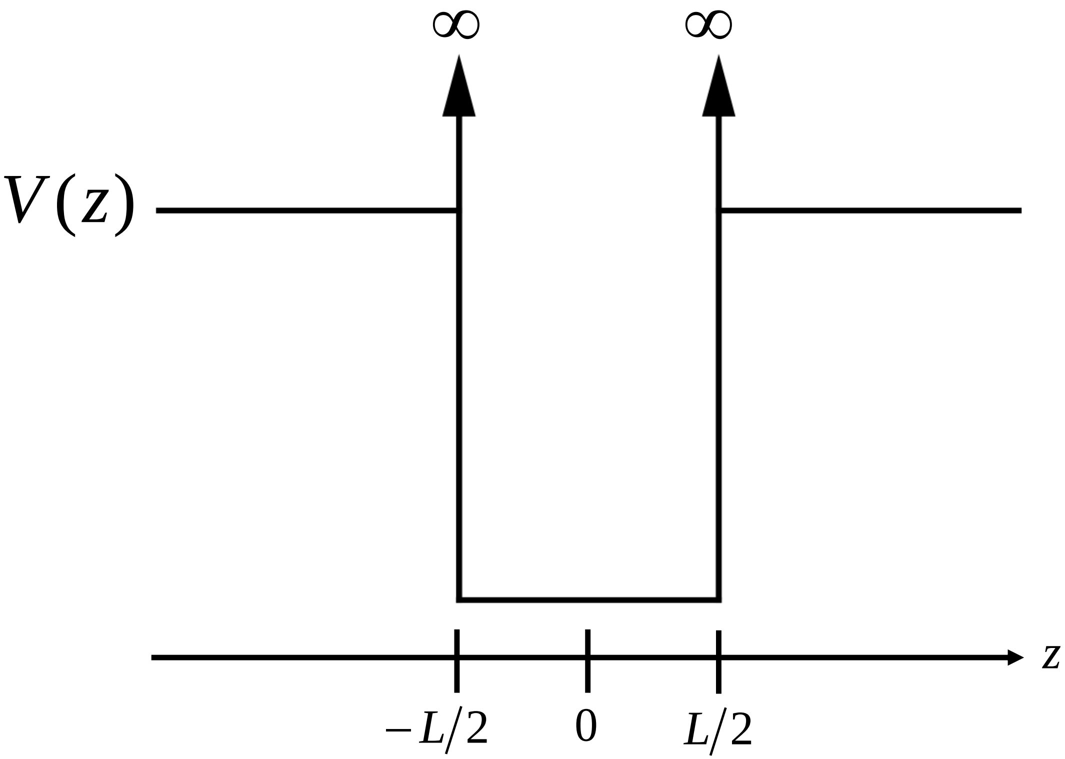
Infinite potential well



$$V(z) = \begin{cases} 0 & \text{for } z < |L/2| \\ \infty & \text{for } z > |L/2| \end{cases}$$

Separation of variables

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2+V(x,y,z)\right]\psi(x,y,z)=E\psi(x,y,z)$$

$$\psi(x,y,z)=\phi_x(x)\phi_y(y)\phi_z(z)$$

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2\phi_x}{\partial x^2}+V(x)\phi_x=E_x\phi_x \qquad -\frac{\hbar^2}{2m^*}\frac{\partial^2\phi_y}{\partial y^2}+V(y)\phi_y=E_y\phi_y \qquad -\frac{\hbar^2}{2m^*}\frac{\partial^2\phi_z}{\partial z^2}+V(z)\phi_z=E_z\phi_z$$

If $V(x,y,z)=V(z)$

$$\psi(x,y,z)=\phi_x(x)\phi_y(y)\phi_z(z)$$

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2\phi_x}{\partial x^2}=E_x\phi_x$$

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2\phi_y}{\partial y^2}=E_y\phi_y$$

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2\phi_z}{\partial z^2}+V(z)\phi_z=E_z\phi_z$$

$$\phi_x=Ae^{ik_x x}$$

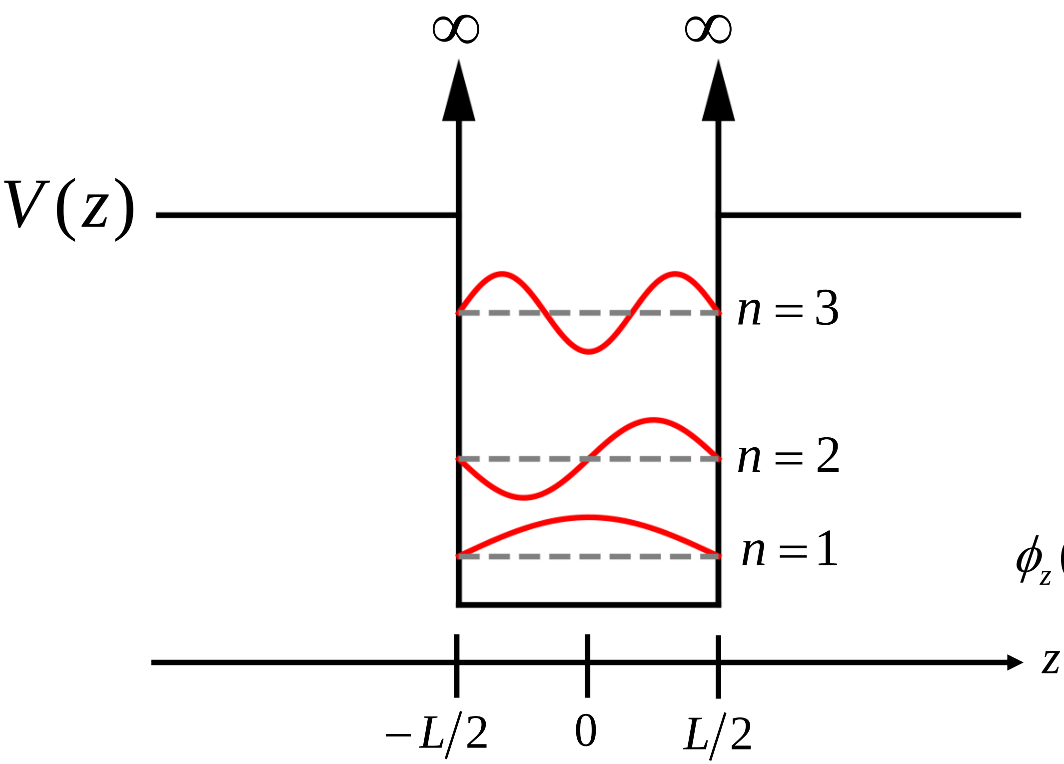
$$\phi_y=Be^{ik_y y}$$

$$\phi_t=\phi_x\phi_y=Ce^{i(k_x x+k_y y)}$$

$$1=\int|\phi_t|^2\,dxdy\rightarrow C=\frac{1}{\sqrt{A}}$$

$$\phi_t=\frac{1}{\sqrt{A}}e^{i(k_x x+k_y y)}$$

Infinite potential well



$$\psi(x,y,z)=\phi_t(x,y)\phi_z(z)$$

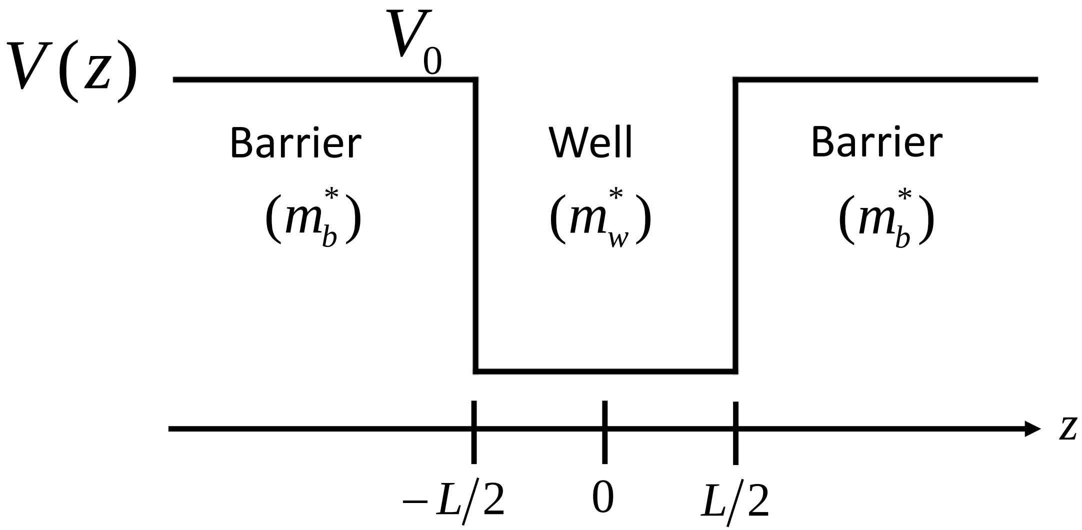
$$\phi_t=\frac{1}{\sqrt{A}}e^{i(k_x x+k_y y)}$$

$$\phi_z(z)=\begin{cases}\sqrt{\frac{2}{L}}\cos\left(\frac{n\pi}{L}z\right) & n=1,3,5,\ldots\\ \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}z\right) & n=2,4,6,\ldots\end{cases}$$

$$E=\frac{\hbar^2}{2m_w^*}\left[k_x^2+k_y^2+\left(\frac{n\pi}{L}\right)^2\right]$$

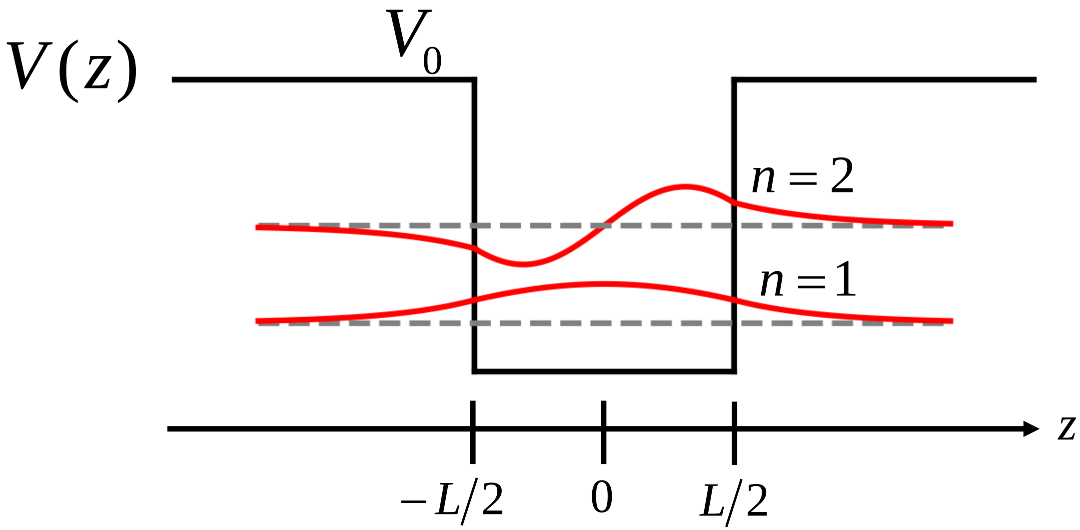
$$V(z)=\begin{cases}0 & \text{for } z<|L/2|\\ \infty & \text{for } z>|L/2|\end{cases}$$

Finite potential well



$$V(z) = \begin{cases} 0 & \text{for } z < |L/2| \\ V_0 & \text{for } z > |L/2| \end{cases}$$

Finite potential well



$$V(z) = \begin{cases} 0 & \text{for } z < |L/2| \\ V_0 & \text{for } z > |L/2| \end{cases}$$

$$\psi(x,y,z) = \phi_t(x,y)\phi_z(z)$$

$$\phi_t = \frac{1}{\sqrt{A}} e^{i(k_x x + k_y y)}$$

Barrier solution

$$\phi_z(z) = \begin{cases} Ce^{-\alpha(z-L/2)} & z > \frac{L}{2} \\ Ce^{\alpha(z+L/2)} & z < \frac{L}{2} \end{cases}$$

Well even solution

$$\phi_z(z) = B\cos(k_z z) \qquad \frac{-L}{2} < z < \frac{L}{2}$$

Well odd solution

$$\phi_z(z) = A\sin(k_z z) \qquad \frac{-L}{2} < z < \frac{L}{2}$$

Finite potential well

Barrier solution

$$\phi_z(z) = \begin{cases} Ce^{-\alpha(z-L/2)} & z > \frac{L}{2} \\ Ce^{\alpha(z+L/2)} & z < \frac{L}{2} \end{cases}$$

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Well odd solution

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① Plug into Schrodinger's Equation ⟶

$$\alpha = \frac{\sqrt{(V_0 - E)2m_b^*}}{\hbar}$$

$$k_z = \frac{\sqrt{2m_w^* E}}{\hbar}$$

② Apply boundary conditions

$$\phi_z\left(L^+/2\right) = \phi_z\left(L^-/2\right)$$

$$\frac{1}{m_b^*}\frac{d}{dz}\phi_z\left(L^+/2\right) = \frac{1}{m_w^*}\frac{d}{dz}\phi_z\left(L^-/2\right)$$

⟶

$$\alpha = k_z \frac{m_b^*}{m_w^*}\tan\left(k_z \frac{L}{2}\right) \quad \text{(even)}$$

$$\alpha = -k_z \frac{m_b^*}{m_w^*}\cot\left(k_z \frac{L}{2}\right) \quad \text{(odd)}$$

Finite potential well

Barrier solution

$$\phi_z(z) = \begin{cases} Ce^{-\alpha(z-L/2)} & z > \frac{L}{2} \\ Ce^{\alpha(z+L/2)} & z < \frac{L}{2} \end{cases}$$

Well even solution

$$\phi_z(z) = B\cos(k_z z) \quad \frac{-L}{2} < z < \frac{L}{2}$$

Well odd solution

$$\phi_z(z) = A\sin(k_z z) \quad \frac{-L}{2} < z < \frac{L}{2}$$

③ After rearranging →

$$\left(\alpha' \frac{L}{2}\right)^2 + \left(k_z \frac{L}{2}\right)^2 = \frac{2m_w^* V_0}{\hbar^2}\left(\frac{L}{2}\right)^2$$

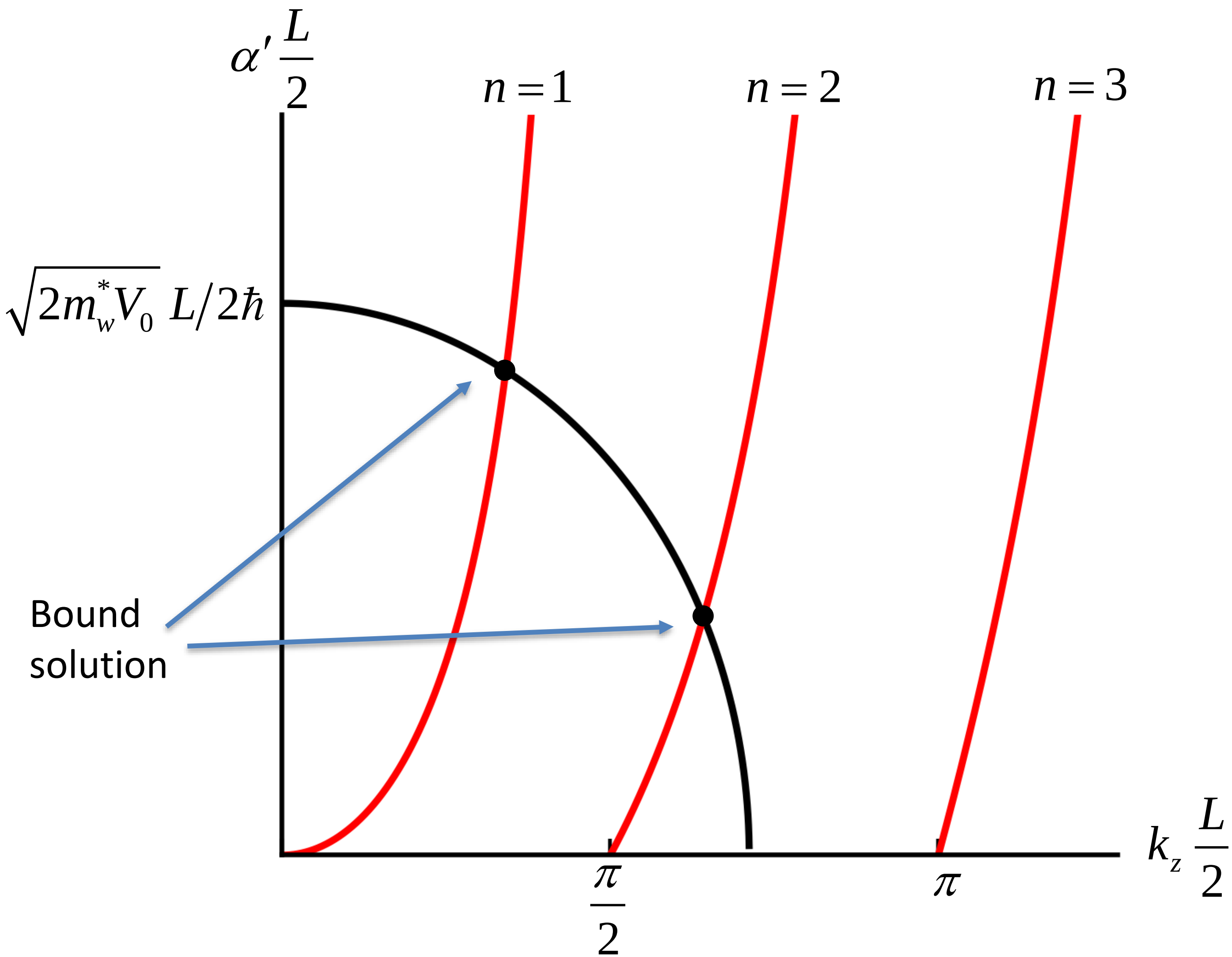
$$\alpha' \frac{L}{2} = k_z \frac{L}{2}\sqrt{\frac{m_b^*}{m_w^*}}\tan\left(k_z \frac{L}{2}\right) \quad \text{(even)}$$

$$\alpha' \frac{L}{2} = -k_z \frac{L}{2}\sqrt{\frac{m_b^*}{m_w^*}}\cot\left(k_z \frac{L}{2}\right) \quad \text{(odd)}$$

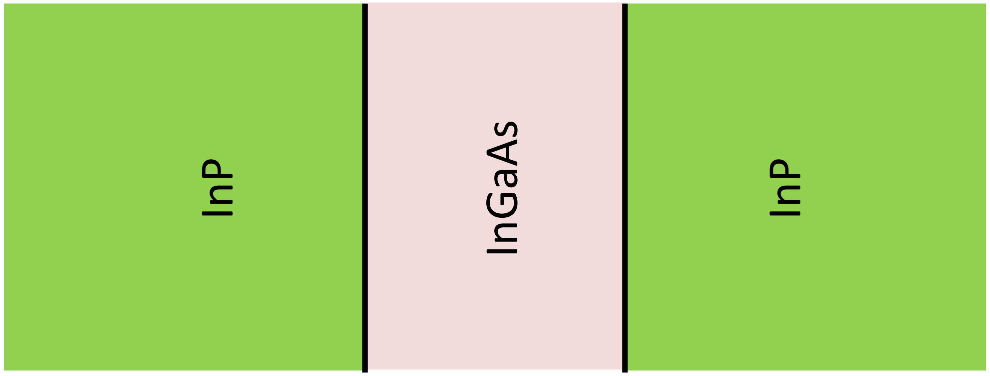
where $\alpha' = \alpha\sqrt{\frac{m_w^*}{m_b^*}}$

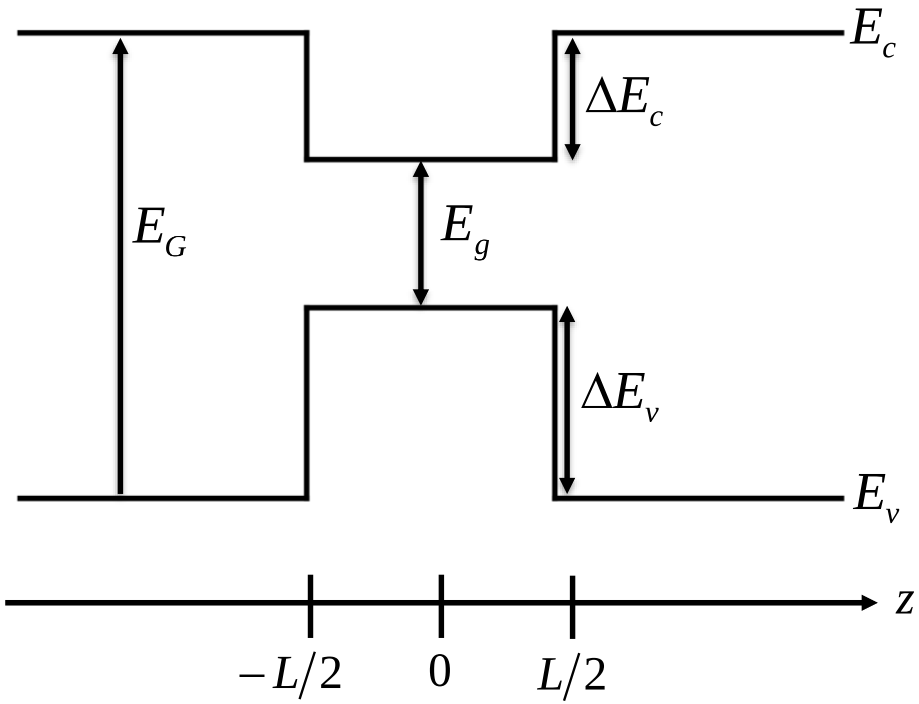
Finite potential well

④ Solve graphically

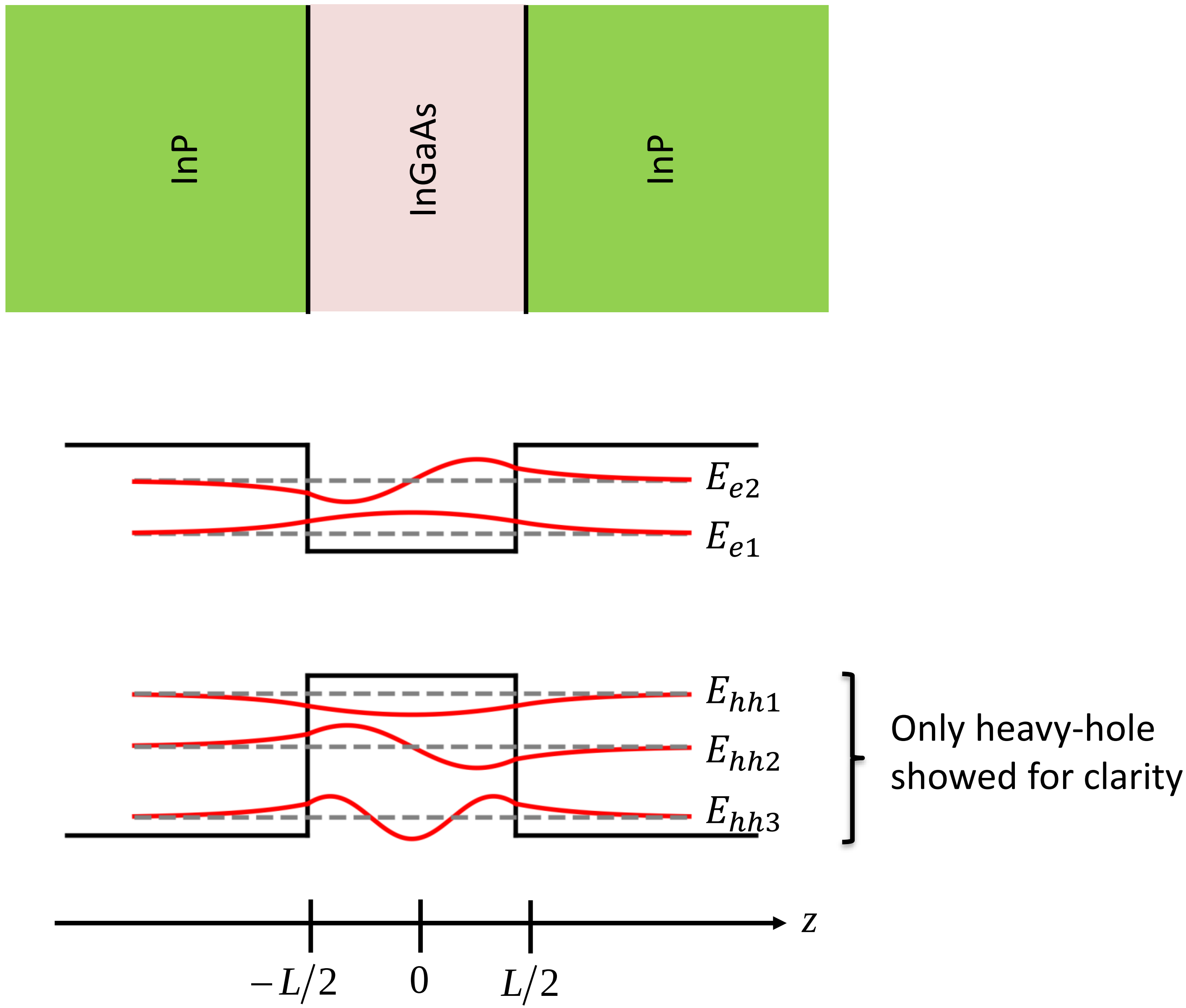


Semiconductor quantum well



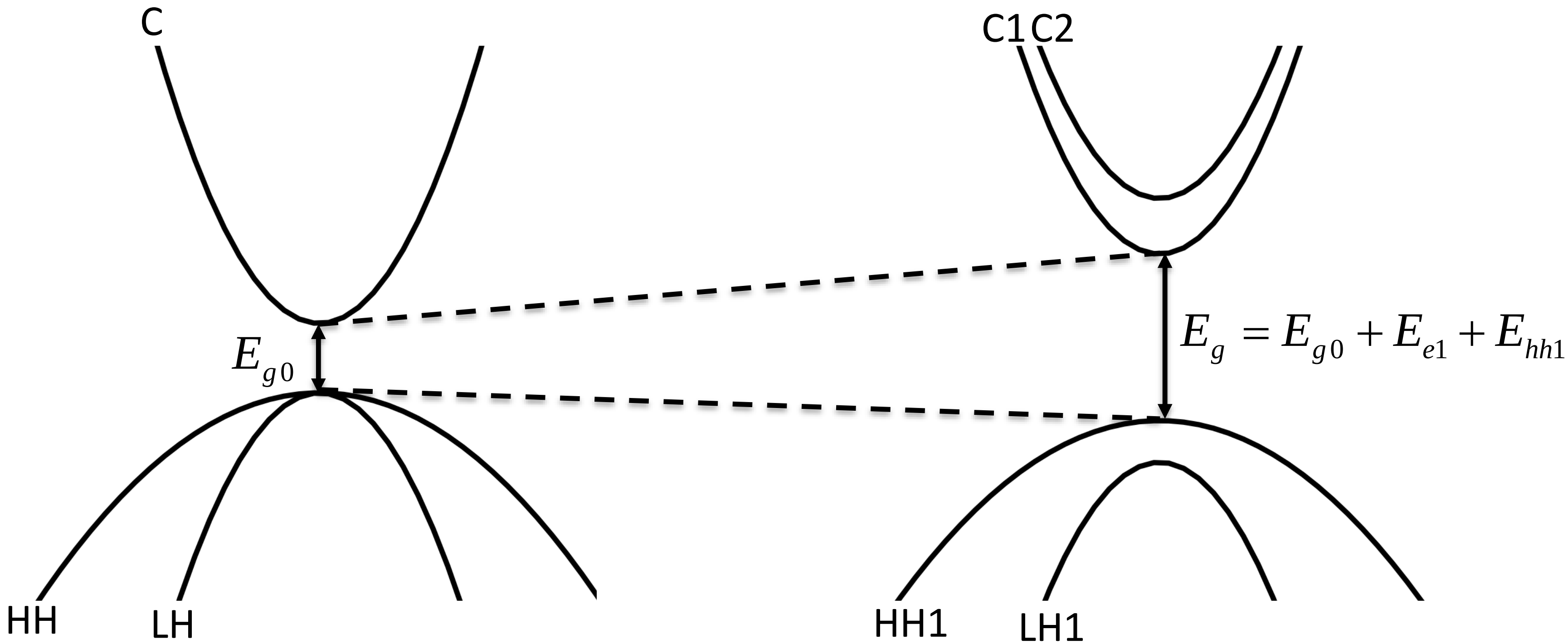


Semiconductor quantum well



Semiconductor quantum well

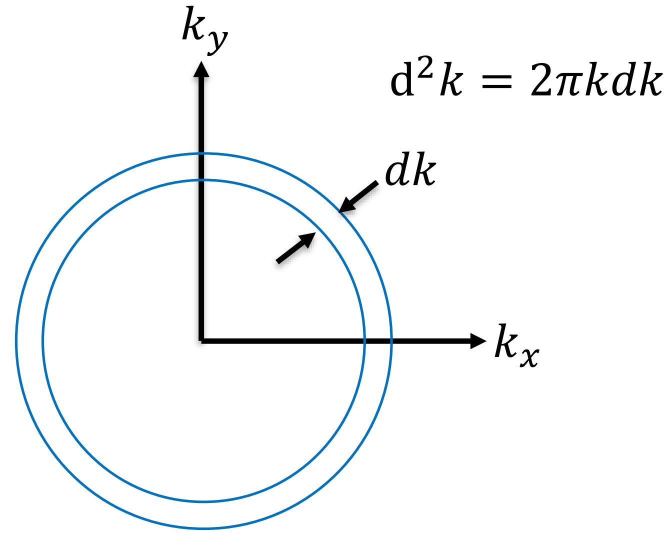
$$E = \frac{\hbar^2}{2m_w^*}\left[k_x^2 + k_y^2 + \left(\frac{n\pi}{L}\right)^2\right]$$



"Bulk" material
No quantum confinement

Quantum well

Density of states (2D)



$$N = 2\int_{-\infty}^{\infty} \frac{1}{(2\pi)^2} d^2\mathbf{k}$$

$$= 2\int_{0}^{\infty} \frac{2\pi k}{(2\pi)^2} dk$$

$$E = \frac{\hbar^2 k^2}{2m_e^*} + E_c + E_{en} \quad \text{(conduction band)}$$

$$k = \sqrt{\frac{2m_e^*(E - E_c - E_{en})}{\hbar^2}}$$

$$\frac{dk}{dE} = \frac{1}{2}\sqrt{\frac{2m_e^*}{\hbar^2}} \frac{1}{\sqrt{E - E_c - E_{en}}}$$

$$N = \int_{E_c + E_{en}}^{\infty} \frac{m_e^*}{\pi\hbar^2} dE$$

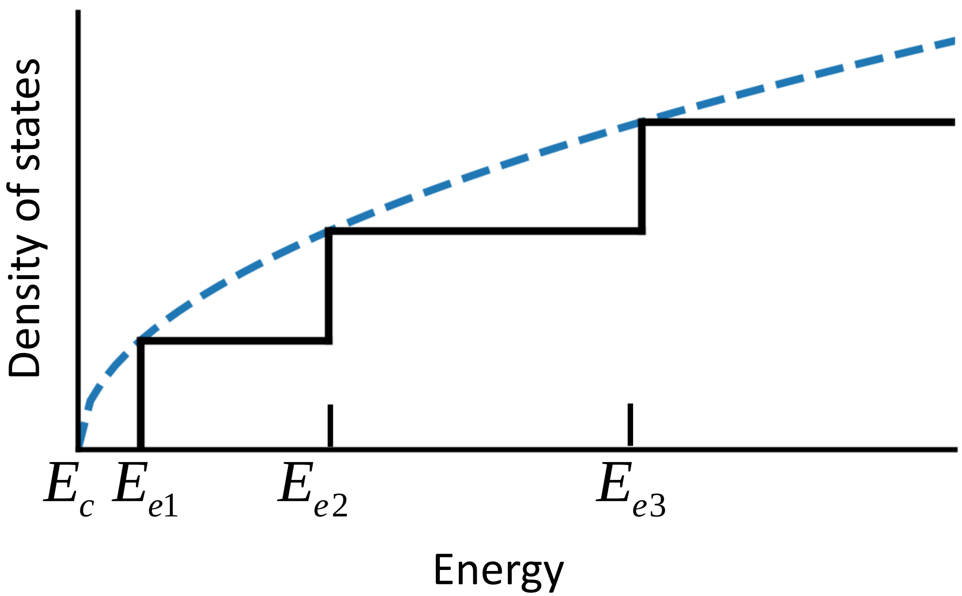
$$= \int_{0}^{\infty} \frac{m_e^*}{\pi\hbar^2} H(E - E_c - E_{en}) dE$$

$$g(E) = \frac{m_e^*}{\pi\hbar^2} H(E - E_c - E_{en})$$

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

(Heaviside step function)

Quantum well density of states



$$g(E) = \begin{cases} 0 & 0 < E - E_c < E_{e1} \\ \dfrac{m_e^*}{\pi\hbar^2} & E_{e1} < E - E_c < E_{e2} \\ 2\dfrac{m_e^*}{\pi\hbar^2} & E_{e2} < E - E_c < E_{e3} \\ 3\dfrac{m_e^*}{\pi\hbar^2} & E_{e3} < E - E_c < E_{e4} \end{cases}$$

(and so on…)

$$g_c(E) = \frac{m_e^*}{\pi\hbar^2} \sum_n H(E - E_c - E_{en})$$